# Global Optimization Techniques in Camera-Robot Calibration: Thesis Defence

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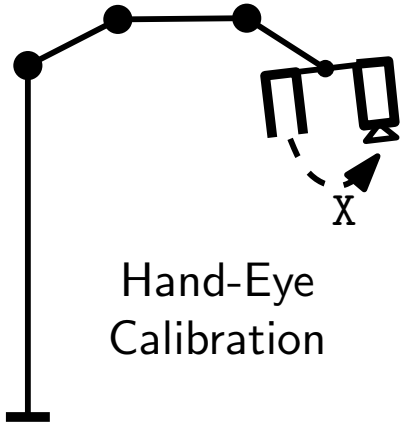
## Camera-Robot Calibration

In order to relate the measurements made by a camera mounted on a robotic gripper to the gripper's coordinate frame, a transformation from the gripper to the camera needs to be determined.



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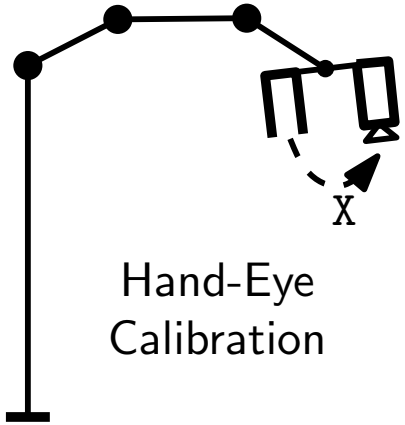
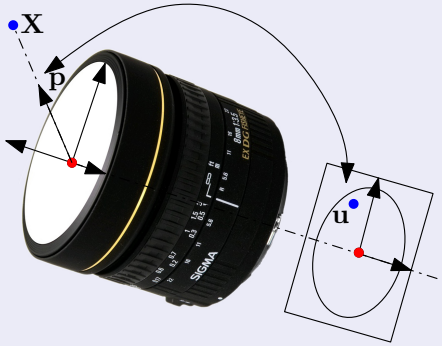
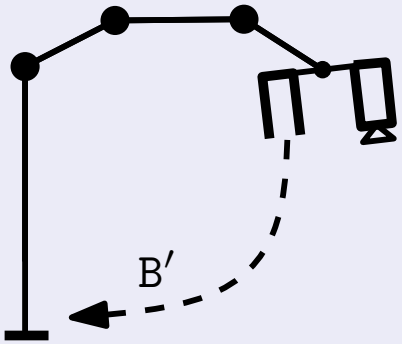


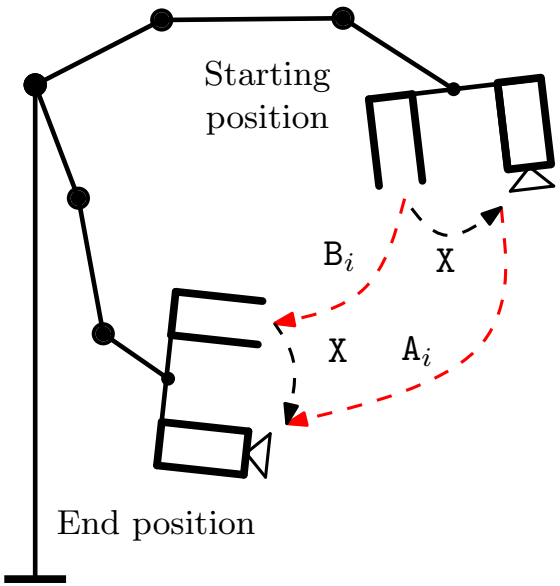

## Camera Calibration




## Robot Calibration



# Hand-Eye Calibration: Classical Approach




- $\mathtt{R}_\mathtt{X}, \mathbf{t}_\mathtt{X}$ separately – Tsai89, Shiu89, Chou91
- $\mathtt{R}_\mathtt{X}, \mathbf{t}_\mathtt{X}$ simultaneously – Houraud95, Daniilidis96
- $\mathtt{R}_\mathtt{X}$ only – Seo09

$$\mathtt{X} = \begin{pmatrix} \mathtt{R}_\mathtt{X} & \mathbf{t}_\mathtt{X} \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

$$\text{Given: } \mathtt{A}_i = \begin{pmatrix} \mathtt{R}_{\mathtt{A}_i} & s\mathbf{t}_{\mathtt{A}_i} \\ \mathbf{0}^\top & 1 \end{pmatrix}, \mathtt{B}_i = \begin{pmatrix} \mathtt{R}_{\mathtt{B}_i} & \mathbf{t}_{\mathtt{B}_i} \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

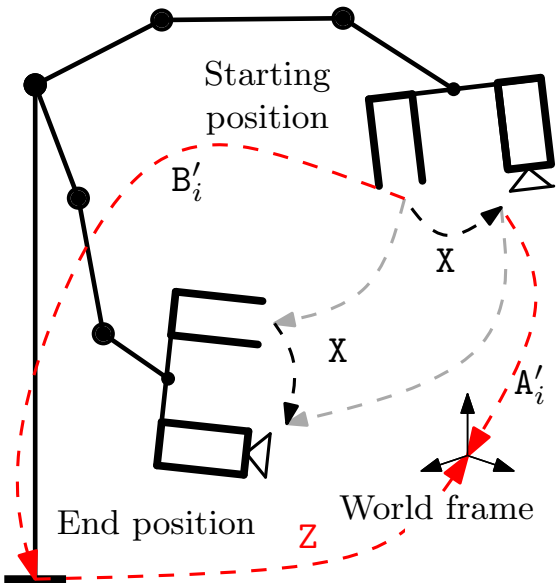
$$\mathtt{A}_i\mathtt{X} = \mathtt{X}\mathtt{B}_i$$

$$i = 1, \ldots, n$$

$$\mathtt{R}_{\mathtt{A}_i}\mathtt{R}_\mathtt{X} = \mathtt{R}_\mathtt{X}\mathtt{R}_{\mathtt{B}_i}$$

$$\mathtt{R}_{\mathtt{A}_i}\mathbf{t}_\mathtt{X} + s\mathbf{t}_{\mathtt{A}_i} = \mathtt{R}_\mathtt{X}\mathbf{t}_{\mathtt{B}_i} + \mathbf{t}_\mathtt{X}$$

# Hand-Eye & Robot-World Calibration: Classical Approach




- Introduced in – Zhuang94
- Quaternions solution – Dornaika98
- Dual quat. – Liu10

$$\mathtt{X} = \begin{pmatrix} \mathtt{R_X} & \mathbf{t}_\mathtt{X} \\ \mathbf{0}^\top & 1 \end{pmatrix}, \mathtt{Z} = \begin{pmatrix} \mathtt{R_Z} & \mathbf{t}_\mathtt{Z} \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

$$\text{Given: } \mathtt{A}'_i = \begin{pmatrix} \mathtt{R}_{\mathtt{A}'_i} & s\mathbf{t}_{\mathtt{A}'_i} \\ \mathbf{0}^\top & 1 \end{pmatrix}, \mathtt{B}'_i = \begin{pmatrix} \mathtt{R}_{\mathtt{B}'_i} & \mathbf{t}_{\mathtt{B}'_i} \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

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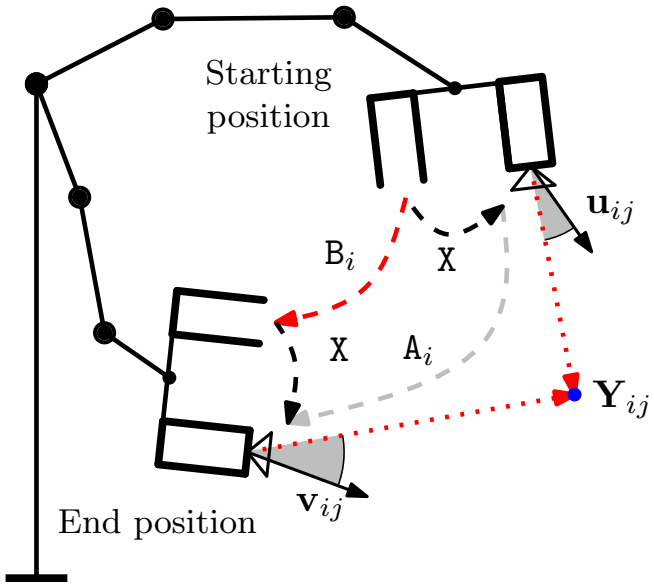
# Thesis Goals




- *Apply global optimization techniques* to the problems of robot-world calibration
- *Use image measurements directly* (instead of using them as a pre-step for computing absolute or relative camera poses $\mathtt{A}_i$)
- *Use geometrically meaningful error measures* and to provide novel insights into these problems
- Show that globally optimal solutions, albeit based on different error measures, *can still be recovered even in situations where the image measurements are not available*

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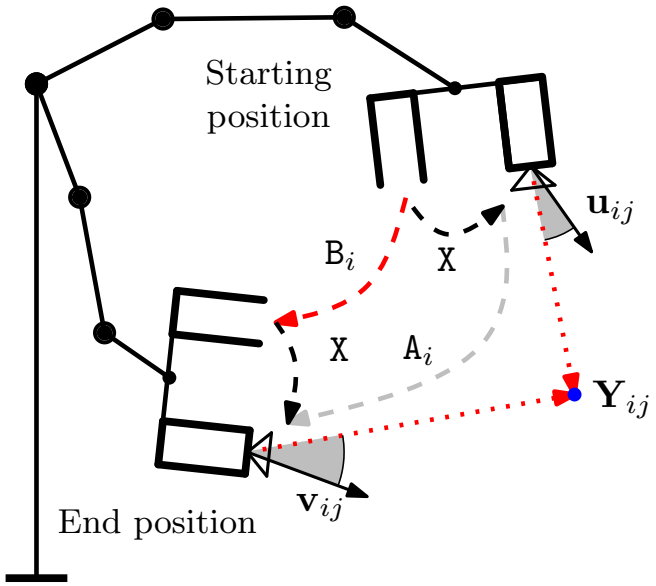


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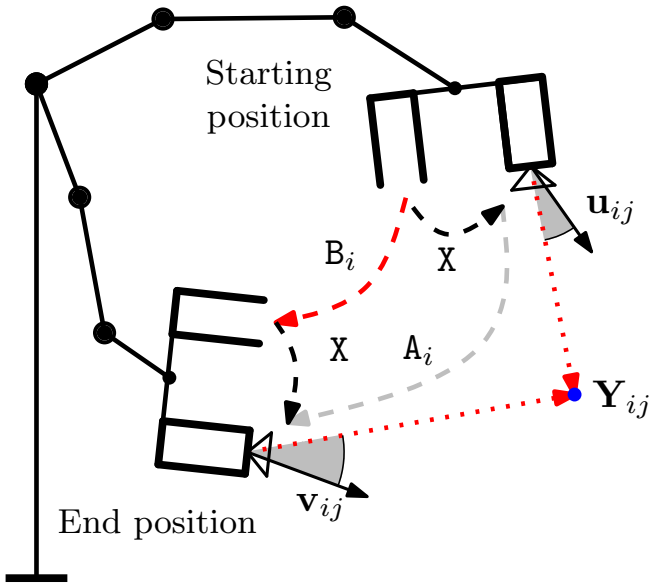


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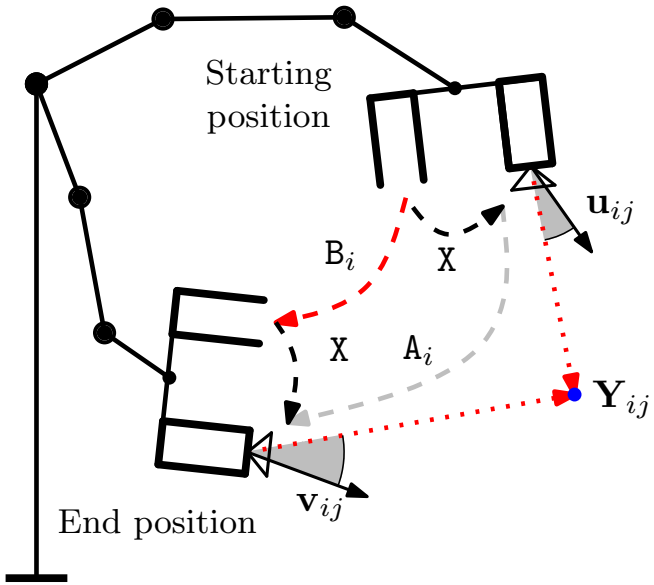


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# Hand Eye Calibration Using Structure-from-Motion

## CVPR 2011—Globally Optimal $\mathtt{t}_\mathtt{X}$

- Based on F. Kahl and R. Hartley, PAMI 2008 (SOCP + bisection)
- Feasibility test based on SOCP and reprojection error
- Recovered $\mathbf{t}_\mathtt{X}$ is optimal in $L_\infty$-norm and computed from image correspondences directly
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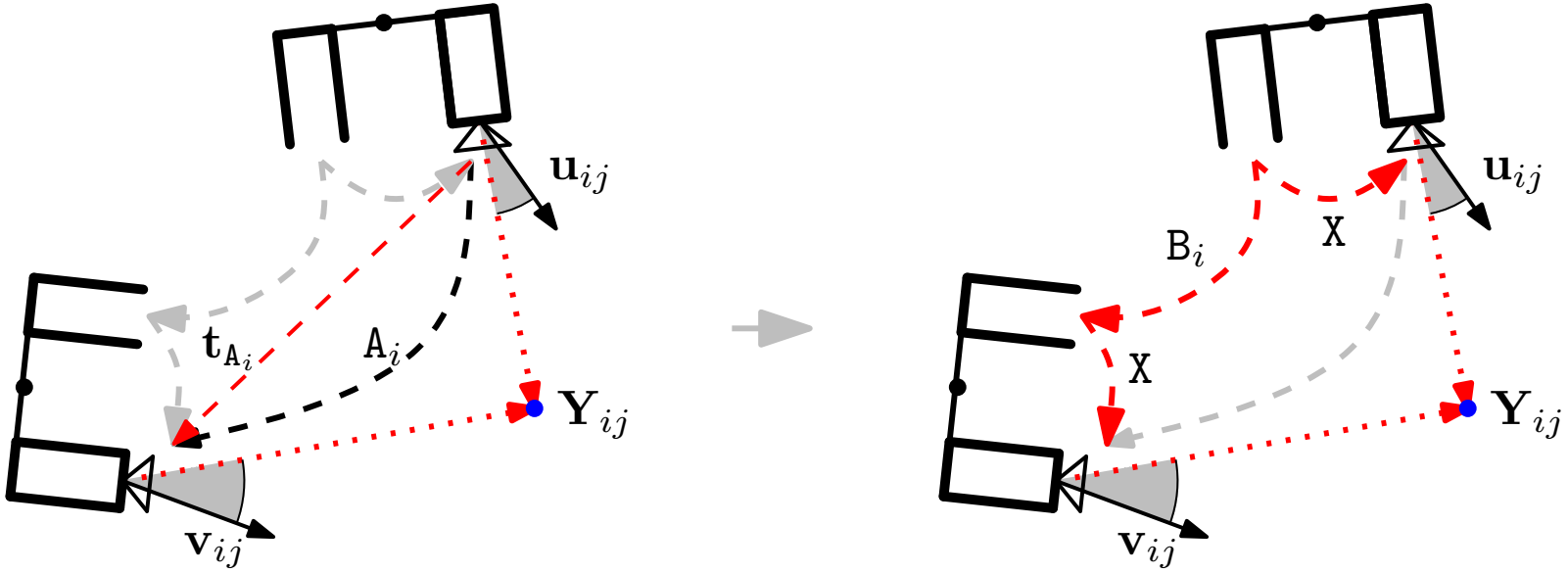
- No need for a calibration target ($(s)\mathbf{t}_{\mathtt{A}_i}$ never used)



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\begin{aligned}
\text{Given } & \mathtt{R}_\mathtt{X}, \mathtt{R}_{\mathtt{B}_i}, \mathbf{t}_{\mathtt{B}_i}, \gamma, \mathbf{u}_{ij}, \mathbf{v}_{ij} \\
\text{do there exist } & \mathbf{t}_\mathtt{X}, \mathbf{Y}_{ij} \\
\text{subject to } & \angle\left(\mathbf{u}_{ij}, \mathbf{Y}_{ij}\right) \leq \gamma \\
& \angle\Big(\mathbf{v}_{ij}, \mathtt{R}_\mathtt{X}\mathtt{R}_{\mathtt{B}_i}\mathtt{R}_\mathtt{X}^\top \mathbf{Y}_{ij} + \\
& \left(\mathtt{I} - \mathtt{R}_\mathtt{X}\mathtt{R}_{\mathtt{B}_i}\mathtt{R}_\mathtt{X}^\top\right)\mathbf{t}_\mathtt{X} + \mathtt{R}_\mathtt{X}\mathbf{t}_{\mathtt{B}_i}\Big) \leq \gamma \\
\text{for } & i = 1, \ldots, n,\ j = 1, \ldots, m\,?
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*Algorithm*

- Estimate the relative camera rotations $\mathtt{R}_{\mathtt{A}_i}$ using SfM
- Estimate $\mathtt{R}_\mathtt{X}$ using $\mathtt{R}_{\mathtt{A}_i}$ and $\mathtt{R}_{\mathtt{B}_i}$
- Use the proposed alg. to find the optimal $\mathbf{t}_\mathtt{X}$ using $\mathtt{R}_\mathtt{X}$ up to required precision $\epsilon$.

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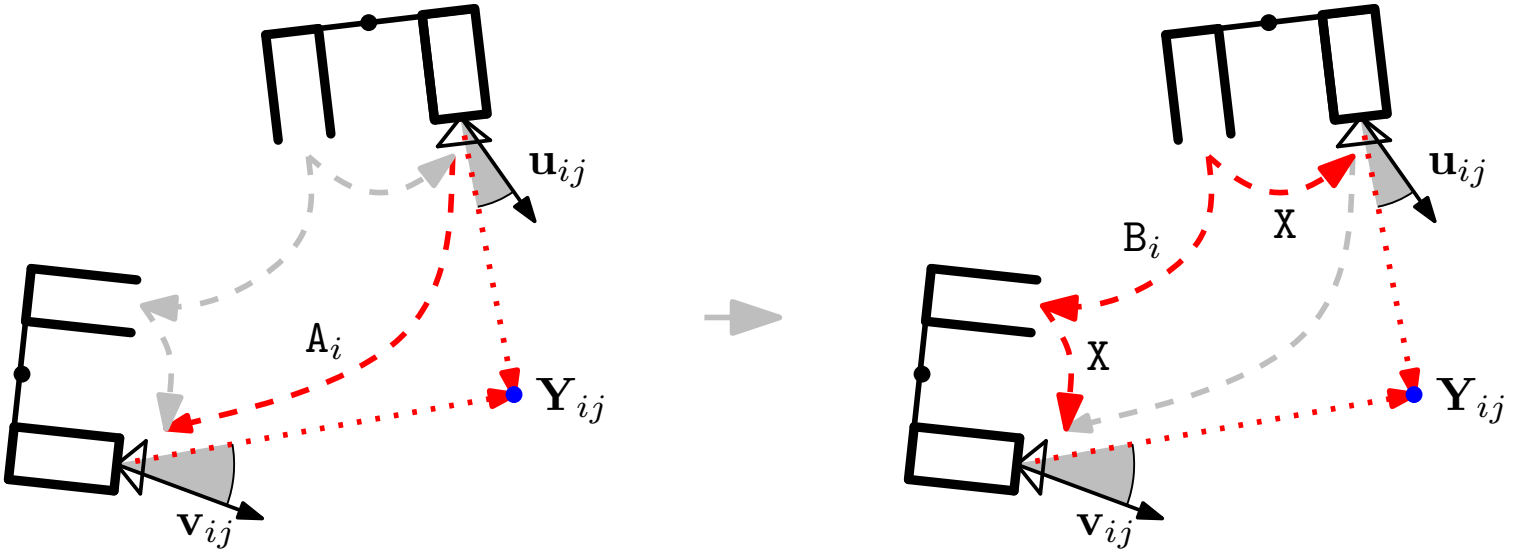

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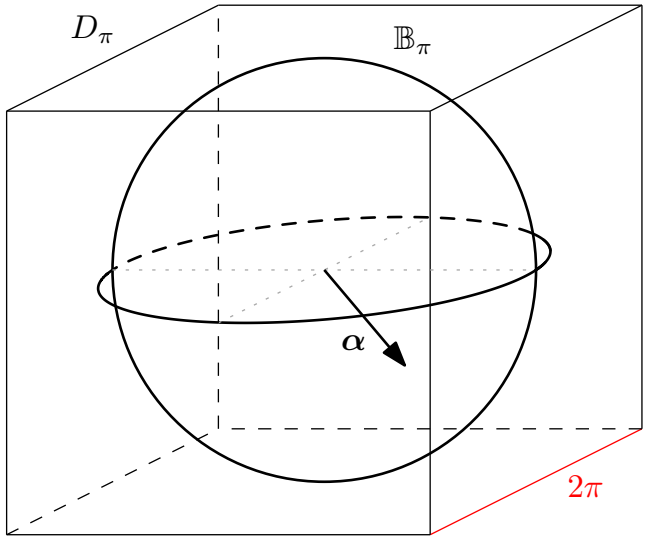


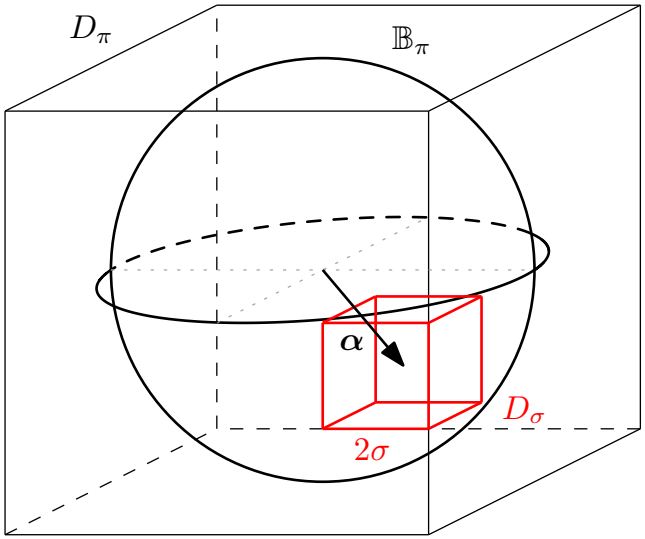

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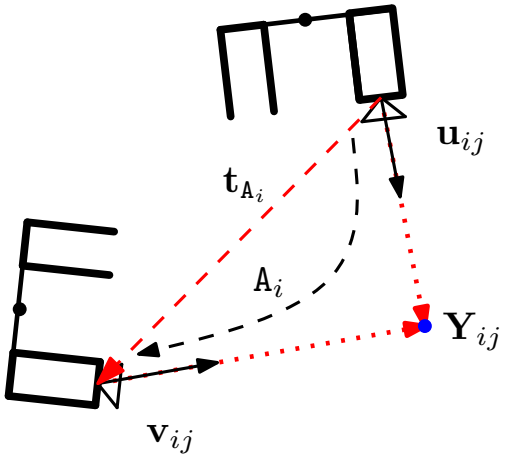
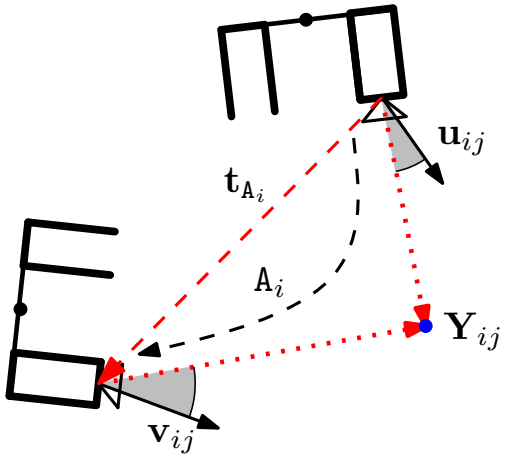
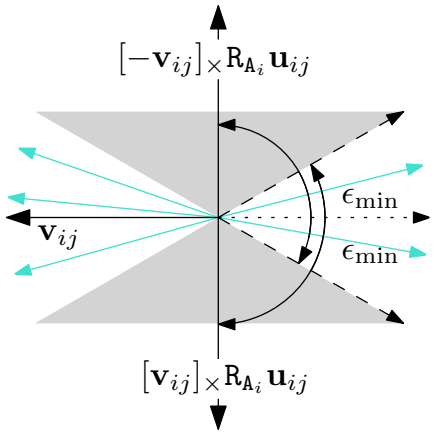



$$\mathbf{t}_{\mathtt{A}_i}\left[\mathbf{v}_{ij}\right]_\times \mathtt{R}_{\mathtt{A}_i}\mathbf{u}_{ij} = 0$$




$$\mathbf{t}_{\mathtt{A}_i}\left[\mathbf{v}_{ij}\right]_\times \mathtt{R}_{\mathtt{A}_i}\mathbf{u}_{ij} \leq e_{ij}$$



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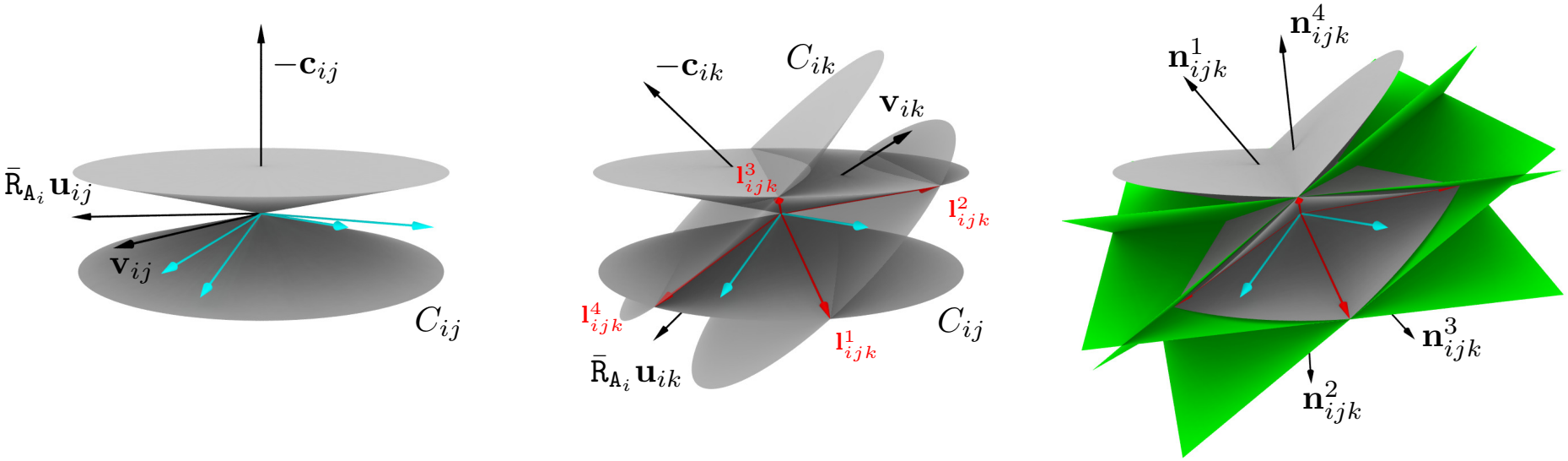

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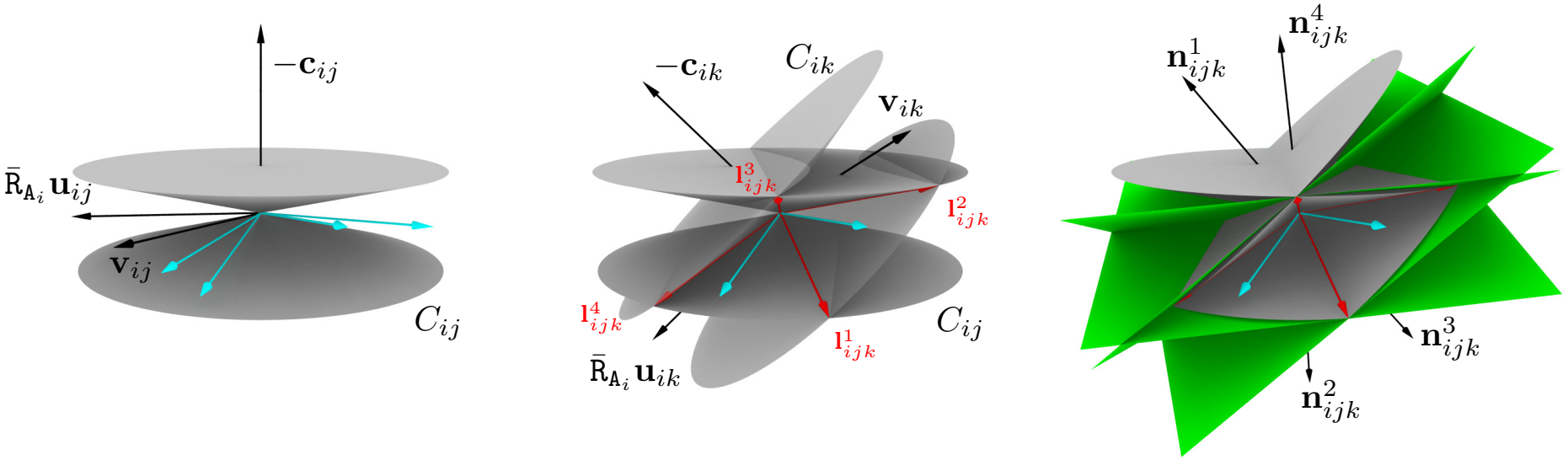

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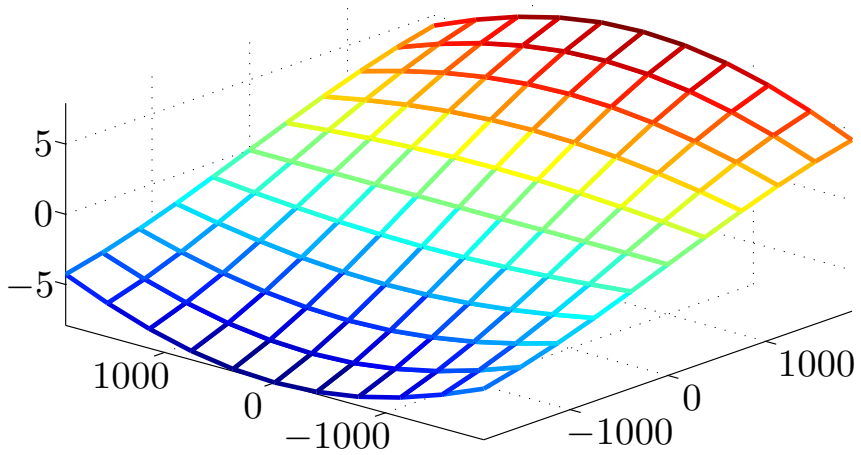
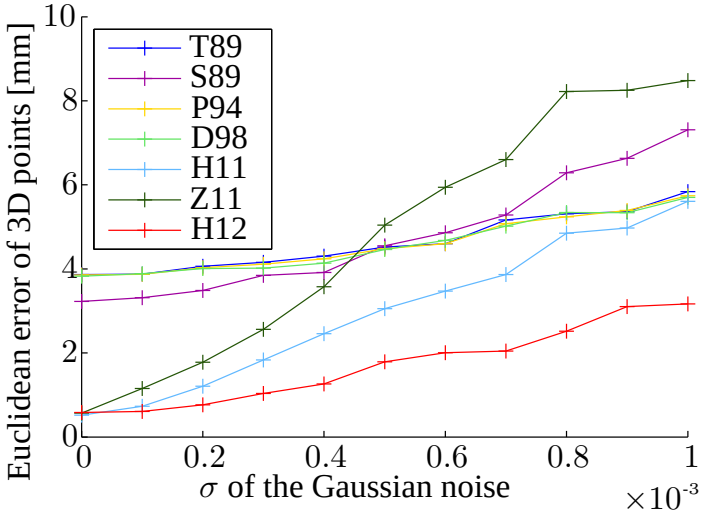
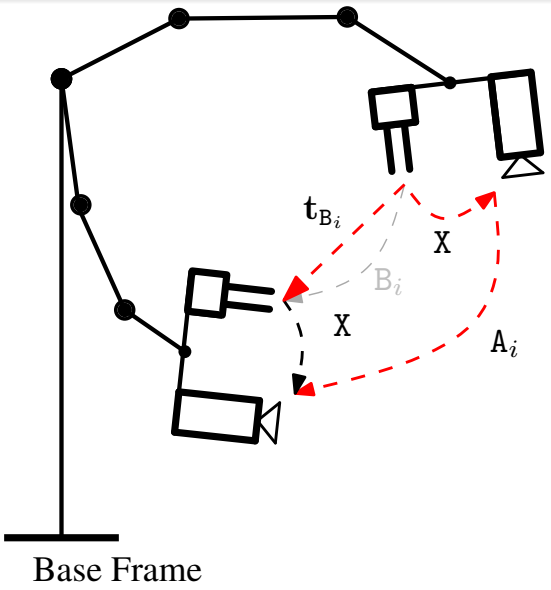


# Hand-Eye Calibration without Hand Orientation

## ACCV 2012

- Variation of hand-eye calibration: HEC w/o known hand rotation
- Arises when the robot is not calibrated or the information from the robot is not available (and the external measurement device provides translation measurements only)
- Problem formulated as a system of 7 equations in 7 unknowns
- Solved using Groebner basis method (via generator by Kukelova)



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$$
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\text{Given } & \mathtt{R}_{\mathtt{A}_i}, \mathtt{R}_{\mathtt{A}_j}, \mathbf{t}_{\mathtt{A}_i}, \mathbf{t}_{\mathtt{A}_j}, \mathbf{t}_{\mathtt{B}_i}, \mathbf{t}_{\mathtt{B}_j} \\
\text{find } & \mathtt{R}_{\mathtt{X}} \in SO(3), \mathbf{t}_{\mathtt{X}} \in \mathbb{R}^3 \\
\text{subject to } & \mathtt{R}_{\mathtt{A}_i}\mathbf{t}_{\mathtt{X}} + \mathbf{t}_{\mathtt{A}_i} = \mathtt{R}^q_{\mathtt{X}}\mathbf{t}_{\mathtt{B}_i} + \mathbf{t}_{\mathtt{X}}, \\
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& a^2 + b^2 + c^2 + d^2 = 1.
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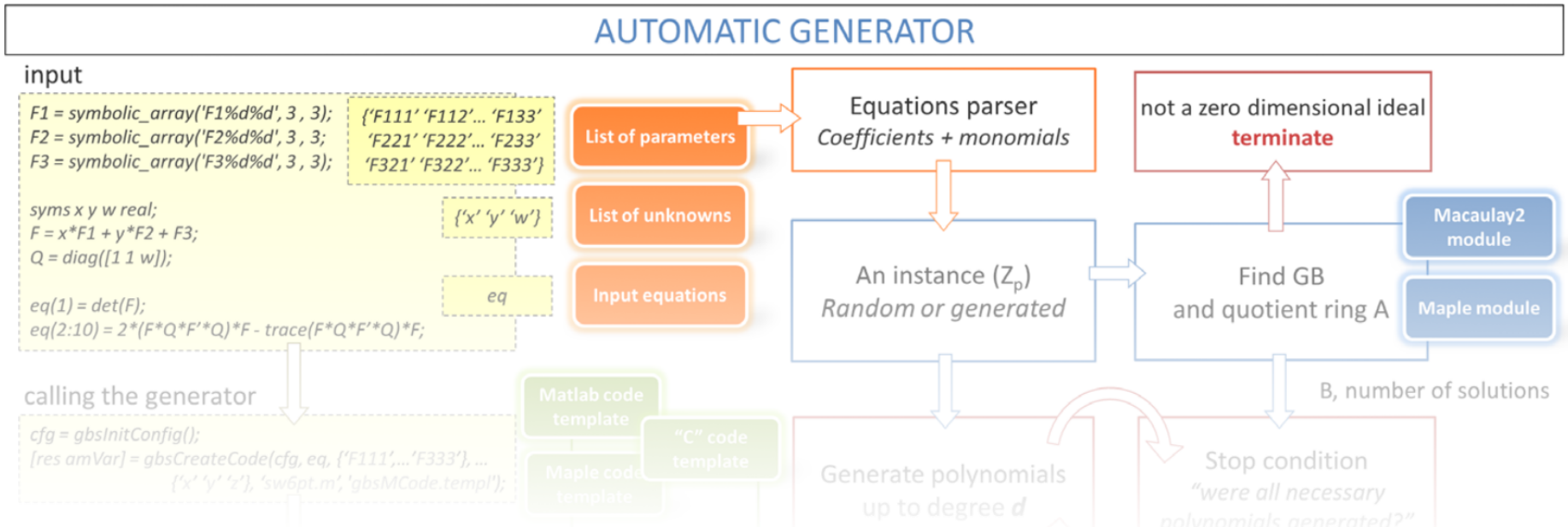
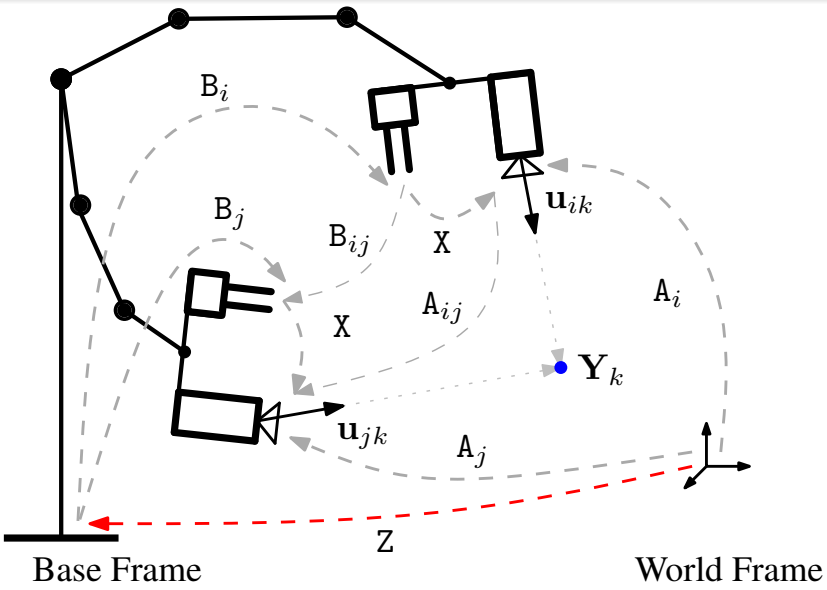


# Robot-World Calibration by LMI Relaxations

## 3DV 2014

- Robot-World Calibration Z
  (with known robot poses, 2D-3D corresp., and hand-eye calibration X)
- Formulated as a bundle adjustment problem
- Polynomial optimization problem of deg. 4 in 4 unknowns ($\mathbf{q}_{\mathtt{Z}}$) of constant size (+ back-substitution for $\mathbf{t}_{\mathtt{Z}}$)
- Solved globally optimally using LMI relaxation method



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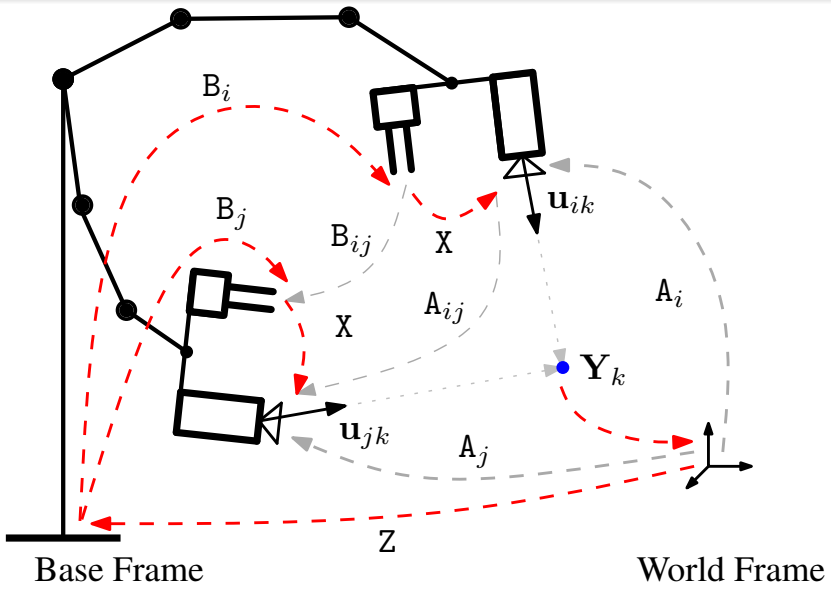

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$$\begin{aligned} \min\ & F(\mathtt{Z}) = \textstyle\sum_{i=1}^{n}\sum_{j=1}^{m} \|\mathtt{C}_{ij}\mathbf{a}_i\|^2 \\ \text{s.t. } & \mathtt{R}_{\mathtt{Z}} \in SO(3), \\ & \mathbf{t}_{\mathtt{Z}} \in \mathbb{R}^3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \min\ & F'(\mathbf{q}_{\mathtt{Z}}) = \mathbf{m}^\top(\mathtt{Q}^\top \mathtt{E}\,\mathtt{Q})\,\mathbf{m} \\ & \qquad\qquad = \mathbf{m}^\top \mathtt{E}'^\top \mathbf{m} \\ \text{s.t. } & q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1, \\ & q_1 \geq 0. \end{aligned}$$

# Robot-World Calibration by LMI Relaxations

## 3DV 2014

- Robot-World Calibration $\mathtt{Z}$
  (with known robot poses, 2D-3D corresp., and hand-eye calibration $\mathtt{X}$)
- Formulated as a bundle adjustment problem
- Polynomial optimization problem of deg. 4 in 4 unknowns ($\mathbf{q}_{\mathtt{Z}}$) of constant size (+ back-substitution for $\mathbf{t}_{\mathtt{Z}}$)
- Solved globally optimally using LMI relaxation method

Linear Matrix Inequalities Relaxation

Polynomial Optimization Problem

$$\begin{aligned} \text{minimize} \quad & p(\mathbf{x}) \\ \text{subject to} \quad & g_i(\mathbf{x}) \geq 0,\ i = 1, \ldots, k, \\ \text{where} \quad & \mathbf{x} = (x_1, x_2, \ldots, x_m)^\top \in \mathbb{R}^m, \\ & p(\mathbf{x}), g_i(\mathbf{x}) \in \mathbb{R}[\mathbf{x}]. \end{aligned}$$

$\Rightarrow$

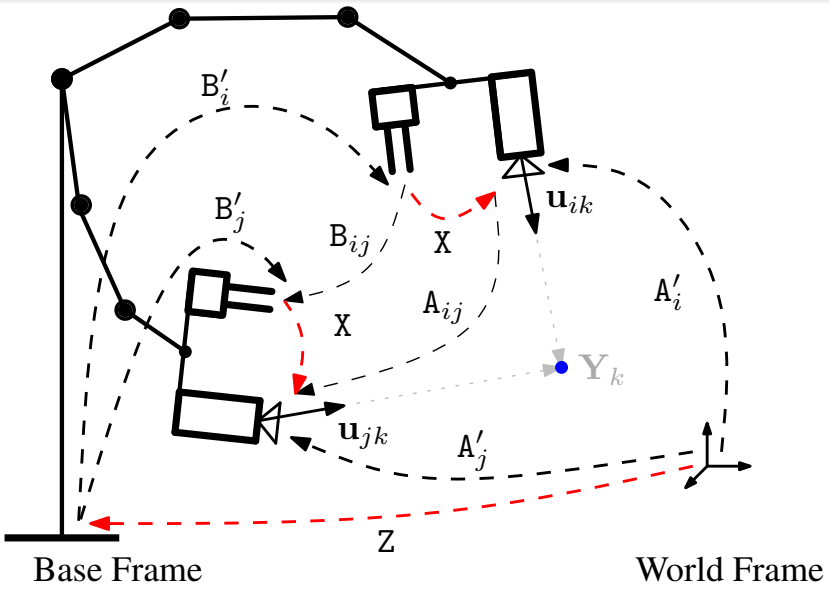
Semidefinite Programming Problem

$$\begin{aligned} \text{minimize} \quad & \mathcal{L}_{\mathbf{y}}(p(\mathbf{x})) \\ \text{subject to} \quad & \mathtt{M}_{\delta}(\mathbf{y}) \succeq 0, \\ & \mathtt{M}_{\delta - d_i}(\mathtt{G}_i, \mathbf{y}) \succeq 0,\ i = 1, \ldots, k, \\ \text{where} \quad & \mathbf{y} = (y_1, y_2, \ldots, y_d)^\top \in \mathbb{R}^d, \end{aligned}$$

# Hand-Eye and Robot-World Calibration by LMI Relaxations

## ICRA 2014

- Revisited classical hand-eye and robot-world calibration formulation (known $\mathtt{B}'_i$, $\mathtt{A}'_i$, corresp. not available)
- Proposed 3 H-E and 3 H-E & R-W formulations based on rotation matrices, quaternions and dual quaternions
- All formulations lead to polynomial optimization problems (POP)
- All problems solved globally optimally using LMI relaxation approach



# Hand-Eye and Robot-World Calibration by LMI Relaxations

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- Revisited classical hand-eye and robot-world calibration formulation (known $\mathtt{B}'_i$, $\mathtt{A}'_i$, corresp. not available)
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$$\begin{aligned} \min \quad & f_1(\mathbf{u}_\mathtt{X}, \mathbf{v}_\mathtt{X}, \mathbf{t}_\mathtt{X}) = \\ & = \textstyle\sum_{i=1}^n \|\mathtt{A}_i\mathtt{X}(\mathbf{u}_\mathtt{X}, \mathbf{v}_\mathtt{X}, \mathbf{t}_\mathtt{X}) - \mathtt{X}(\mathbf{u}_\mathtt{X}, \mathbf{v}_\mathtt{X}, \mathbf{t}_\mathtt{X})\mathtt{B}_i\|^2 \\ \text{s.t.} \quad & \mathbf{u}_\mathtt{X}^\top \mathbf{u}_\mathtt{X} = 1, \mathbf{v}_\mathtt{X}^\top \mathbf{v}_\mathtt{X} = 1, \mathbf{u}_\mathtt{X}^\top \mathbf{v}_\mathtt{X} = 0. \end{aligned}$$

$$\begin{aligned} \min \quad & f_2(\mathbf{q}_\mathtt{X}, \mathbf{t}_\mathtt{X}) = \\ & = \textstyle\sum_{i=1}^n \|\mathtt{A}_i\mathtt{X}(\mathbf{q}_\mathtt{X}, \mathbf{t}_\mathtt{X}) - \mathtt{X}(\mathbf{q}_\mathtt{X}, \mathbf{t}_\mathtt{X})\mathtt{B}_i\|^2 \\ \text{s.t.} \quad & \mathbf{q}_\mathtt{X}^\top \mathbf{q}_\mathtt{X} = 1, q_{\mathtt{X}1} \geq 0. \end{aligned}$$

$$\begin{aligned} \min \quad & f_3(\hat{\mathbf{q}}_\mathtt{X}) = \textstyle\sum_{i=1}^n \|\hat{\mathbf{a}}_i \otimes \hat{\mathbf{q}}_\mathtt{X} - \hat{\mathbf{q}}_\mathtt{X} \otimes \hat{\mathbf{b}}_i\|^2 \\ \text{s.t.} \quad & \mathbf{q}_\mathtt{X}^\top \mathbf{q}_\mathtt{X} = 1, \\ & q_{\mathtt{X}1}q_{\mathtt{X}5} + q_{\mathtt{X}2}q_{\mathtt{X}6} + q_{\mathtt{X}3}q_{\mathtt{X}7} + q_{\mathtt{X}4}q_{\mathtt{X}8} = 0, \\ & q_{\mathtt{X}1} \geq 0. \end{aligned}$$

$$\begin{aligned} \min \quad & f_4(\mathbf{u}_\mathtt{X}, \mathbf{v}_\mathtt{X}, \mathbf{t}_\mathtt{X}, \mathbf{u}_\mathtt{Z}, \mathbf{v}_\mathtt{Z}, \mathbf{t}_\mathtt{Z}) = \\ & = \textstyle\sum_{i=1}^m \|\mathtt{A}'_i\mathtt{X}(\mathbf{u}_\mathtt{X}, \mathbf{v}_\mathtt{X}, \mathbf{t}_\mathtt{X}) - \mathtt{Z}(\mathbf{u}_\mathtt{Z}, \mathbf{v}_\mathtt{Z}, \mathbf{t}_\mathtt{Z})\mathtt{B}'_i\|^2 \\ \text{s.t.} \quad & \mathbf{u}_\mathtt{X}^\top \mathbf{u}_\mathtt{X} = 1, \mathbf{v}_\mathtt{X}^\top \mathbf{v}_\mathtt{X} = 1, \mathbf{u}_\mathtt{X}^\top \mathbf{v}_\mathtt{X} = 0, \\ & \mathbf{u}_\mathtt{Z}^\top \mathbf{u}_\mathtt{Z} = 1, \mathbf{v}_\mathtt{Z}^\top \mathbf{v}_\mathtt{Z} = 1, \mathbf{u}_\mathtt{Z}^\top \mathbf{v}_\mathtt{Z} = 0. \end{aligned}$$

$$\begin{aligned} \min \quad & f_5(\mathbf{q}_\mathtt{X}, \mathbf{t}_\mathtt{X}, \mathbf{q}_\mathtt{Z}, \mathbf{t}_\mathtt{Z}) = \\ & = \textstyle\sum_{i=1}^m \|\mathtt{A}'_i\mathtt{X}(\mathbf{q}_\mathtt{X}, \mathbf{t}_\mathtt{X}) - \mathtt{Z}(\mathbf{q}_\mathtt{Z}, \mathbf{t}_\mathtt{Z})\mathtt{B}'_i\|^2 \\ \text{s.t.} \quad & \mathbf{q}_\mathtt{X}^\top \mathbf{q}_\mathtt{X} = 1, q_{\mathtt{X}1} \geq 0, \mathbf{q}_\mathtt{Z}^\top \mathbf{q}_\mathtt{Z} = 1, q_{\mathtt{Z}1} \geq 0. \end{aligned}$$

$$\begin{aligned} \min \quad & f_6(\hat{\mathbf{q}}_\mathtt{X}, \hat{\mathbf{q}}_\mathtt{Z}) = \textstyle\sum_{i=1}^m \|\hat{\mathbf{a}}'_i \otimes \hat{\mathbf{q}}_\mathtt{X} - \hat{\mathbf{q}}_\mathtt{X} \otimes \hat{\mathbf{b}}'_i\|^2 \\ \text{s.t.} \quad & \mathbf{q}_\mathtt{X}^\top \mathbf{q}_\mathtt{X} = 1, q_{\mathtt{X}1}q_{\mathtt{X}5} + q_{\mathtt{X}2}q_{\mathtt{X}6} + q_{\mathtt{X}3}q_{\mathtt{X}7} + q_{\mathtt{X}4}q_{\mathtt{X}8} = 0, \\ & \mathbf{q}_\mathtt{Z}^\top \mathbf{q}_\mathtt{Z} = 1, q_{\mathtt{Z}1}q_{\mathtt{Z}5} + q_{\mathtt{Z}2}q_{\mathtt{Z}6} + q_{\mathtt{Z}3}q_{\mathtt{Z}7} + q_{\mathtt{Z}4}q_{\mathtt{Z}8} = 0, \\ & q_{\mathtt{X}1} \geq 0, q_{\mathtt{Z}1} \geq 0. \end{aligned}$$

# Hand-Eye and Robot-World Calibration by LMI Relaxations

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| Method | Variables | Degree | # Monomials | # Lin. Vars | Time$^1$(s) | Time$^2$(s) |
|---|---|---|---|---|---|---|
| *uvhec* | 9 | 4 | 124 | 715 | 1.45 | 0.45 |
| *qhec* | 7 | 4 | 85 | 330 | 0.48 | 0.29 |
| *dqhec* | 8 | 4 | 177 | 495 | 0.99 | 0.46 |
| *uvherwc* | 18 | 4 | 280 | 7315 | 953.76 | 60.89 |
| *qherwc* | 14 | 4 | 209 | 3060 | 68.38 | 7.91 |
| *dqherwc* | 16 | 2 | 112 | 4845 | 309.25 | 16.85 |

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# GpoSolver: A Matlab/C++ Toolbox for Global Polynomial Optim.

## *Optimization Methods and Software, 2016*

*PMI class*

$$\min\ -x_1^2 - x_2^2 \quad \text{s.t.} \begin{pmatrix} 1 - a_1 x_1 x_2 & x_1 \\ x_1 & a_2 - x_1^2 - x_2^2 \end{pmatrix} \succeq 0$$

*Problem modeling*

*PMI class definition*

```
syms x1 x2 a1 a2 real;
problem.vars = [x1, x2];
problem.ppars = [a1, a2];
problem.obj = -x1^2 - x2^2;
problem.cons = ...
 {[1 - a1*x1*x2, x1; ...
 x1, a2 - x1^2 - x2^2] >= 0};
```

*Code generation*

```
par.relax_order = 2;
par.classname = 'PmiProblem';
par.filename = 'pmi_problem';
gpogenerator(problem, par);
```

pmi_problem.h | pmi_problem.cpp

MATLAB

*Problem solving*

*PMI instance definition*

```
#include <gposolver/gposolver_csdp.h>
#include "pmi_problem.h"
using namespace GpoSolver;
...
GpoSolverCsdp<PmiProblem> gposolver;
GpoSolverStatus status;
GpoSolverSolutions sols;

double as[2];
as[0] = ...
as[1] = ...
status = gposolver.solve(2, NULL, (double*) as, sols);
```

*Compilation & Linking*

+
*Your codebase*
+
pmi_problem.h
pmi_problem.cpp

C++

*PMI instance solving*

pmi_solver.exe

```
as[0] = ...
as[1] = ...
```

http://cmp.felk.cvut.cz/gposolver

# Publications Related to the Thesis

## Impacted journal papers

### TPAMI: IF 5.781

- Jan Heller, Michal Havlena and Tomas Pajdla. Globally Optimal Hand-Eye Calibration Using Branch-and-Bound. *IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI)*, Volume 38 , Issue 5, 2016

### OMS: IF 1.624

- Jan Heller, Tomas Pajdla. GpoSolver: a Matlab/C++ toolbox for global polynomial optimization. *Optimization Methods and Software (OMS)*, Volume 31 , Issue 2, 2016

# Publications Related to the Thesis

## Conference papers

- **Jan Heller**, Tomas Pajdla. World-base calibration by global polynomial optimization. *In 2nd International Conference on 3D Vision (3DV)*, pages 593–600. December, 2014.
- **Jan Heller**, Didier Henrion, and Tomas Pajdla. Hand-eye and robot-world calibration by global polynomial optimization. *In IEEE International Conference on Robotics and Automation (ICRA)*, pages 3157–3164. IEEE, May, 2014.
- Zuzana Kukelova, **Jan Heller**, and Tomas Pajdla. Hand-eye calibration without hand orientation measurement using minimal solution. *In Asian Conference on Computer Vision (ACCV)*, pages 576–589. Springer, November, 2012.
- **Jan Heller**, Michal Havlena, and Tomas Pajdla. A branch-and-bound algorithm for globally optimal hand-eye calibration. *In IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 1608–1615. IEEE, June, 2012.
- **Jan Heller**, Michal Havlena, Akihiro Sugimoto, and Tomas Pajdla. Structure-from-motion based hand-eye calibration using $L_\infty$ minimization. *In IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 3497–3503. IEEE, June, 2011.

# Software Related to the Thesis

## bbhec

- **bbhec**: A Branch-and-Bound Algorithm for Globally Optimal Hand-Eye Calibration

## minhec

- **minhec**: Hand-Eye Calibration without Hand Orientation Measurement Using Minimal Solution

## mpherwc

- **mpherwc**: Hand-Eye and Robot-World Calibration by Global Polynomial Optimization

## GpoSolver

- **GpoSolver**: A MATLAB/C++ Toolbox for Global Polynomial Optimization

# Jan Heller

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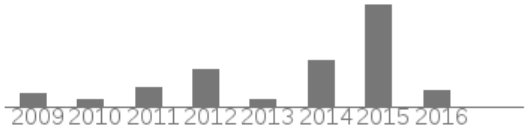


| Title 1–20 | Cited by | Year |
|---|---|---|
| Structure-from-motion based hand-eye calibration using L∞ minimization<br>J Heller, M Havlena, A Sugimoto, T Pajdla<br>Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on ... | 23 | 2011 |
| A branch-and-bound algorithm for globally optimal hand-eye calibration<br>J Heller, M Havlena, T Pajdla<br>Computer Vision and Pattern Recognition (CVPR), 2012 IEEE Conference on ... | 12 | 2012 |
| Stereographic rectification of omnidirectional stereo pairs<br>J Heller, T Pajdla<br>Computer Vision and Pattern Recognition, 2009. CVPR 2009. IEEE Conference on ... | 12 | 2009 |
| Hand-Eye and Robot-World Calibration by Global Polynomial Optimization<br>J Heller, D Henrion, T Pajdla<br>IEEE International Conference on Robotics and Automation | 7 | 2014 |
| CMP SfM Web Service v1. 0<br>J Heller, M Havlena, A Torii, T Pajdla | 5 | 2010 |
| Singly-bordered block-diagonal form for minimal problem solvers<br>Z Kukelova, M Bujnak, J Heller, T Pajdla<br>Computer Vision--ACCV 2014, 488-502 | 4 | 2014 |
| Hand-Eye calibration without hand orientation measurement using minimal solution<br>Z Kukelova, J Heller, T Pajdla<br>Computer Vision–ACCV 2012, 576-589 | 4 | 2012 |
| 3d surface models from opportunity mer navcam<br>M Havlena, M Jančošek, J Heller, T Pajdla<br>EPSC Abstracts 5 | 4 | 2010 |
| Globally optimal hand-eye calibration using branch-and-bound<br>J Heller, M Havlena, T Pajdla<br>IEEE | 3 | 2015 |

**Co-authors** View all...

- Tomas Pajdla
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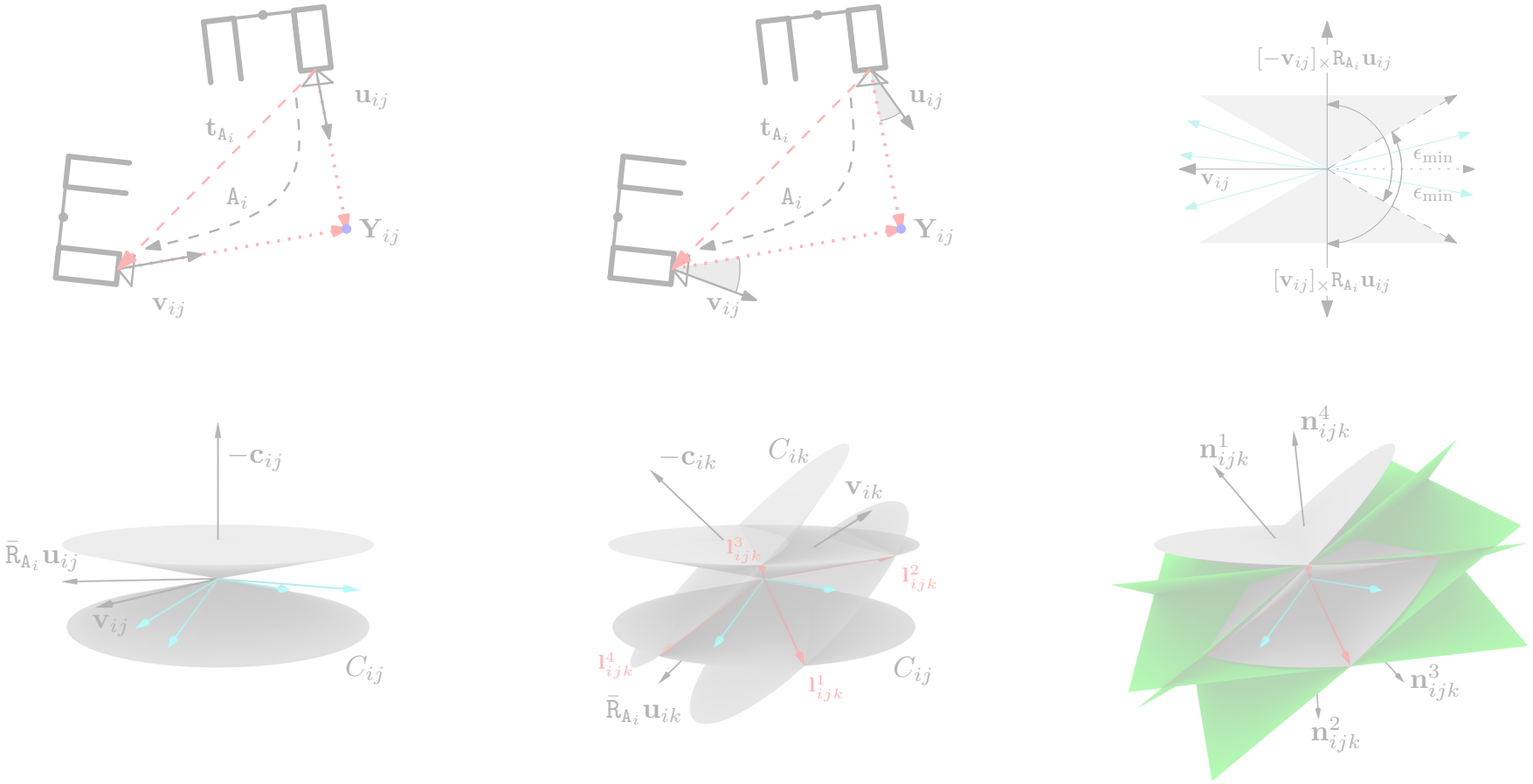
- Zuzana Kukelova
- Martin Bujnak
- Akihiro Sugimoto
- Luc Van Gool
- Andrew Fitzgibbon




Thank you for your attention