ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

fakulta jaderná a fyzikálně inženýrská

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Supervisor's report on the doctoral thesis of Daniel Dombek:

Non-standard representations of numbers

The PhD thesis of Daniel Dombek focuses on non-standard numeration systems from diverse points of view. Main focus is put on representation of real numbers in (both positive and negative) non-integer bases, as considered by Renyi (1957) and Ito and Sadahiro (2009), in particular, with accent to numbers with integer expansions, i.e. such that their representation uses only non-negative powers of the base. Less extensive is the second topic, representation of algebraic integers in a given number field by sums of (distinct) units, or elements of small algebraic norm. The connection between the two seemingly independent topics is well explained, emphasized with the use of combinatorial methods in number theoretical problems.

The thesis is divided into three parts. First is a very detailed introduction to all the notions used with an overview of the field. The second part concentrates on representation of reals in negative base and comparison of Rényi and Ito-Sadahiro numeration systems. Third part is devoted to the so-called unit sum number problem and its generalization. In the second and third part, the author first presents the state of the art which is base for his research problems, and then provides his own results. The presentation of Daniel Dombek is very clear and self-contained with sufficient emphasis on both mathematical precision and explanation on examples.

During his PhD studies, Daniel Dombek was working with great enthusiasm. He always wanted to understand the problems into the tiniest detail and with all the context and did not hesitate to tackle even very technical work. This gave rise to several interesting results, which are described in the presented thesis. Among the most important

contribution is the description of the antimorphism generating numbers with integer expansion in base $-\beta$. and its relation to the canonical morphism playing similar role for positive base β.

The results of the thesis have been assembled in six papers authored or co-authored by Daniel Dombek. Five of them have been already published or accepted for publication in refereed scientific journals focused on theoretical computer science or number theory. On some of the papers, Daniel Dombek is the only author, others arised from international collaboration with several different coauthors. This proves Daniel's capacity of both independent research and team work. The contributions of Dombek has also been accepted for presentation at several international conferences, such as Words 2011 and 2013, CANT 2012, Journées Arithmétiques 2011 and 2013, Numeration and Substitution 2014. This altogether testifies about the scientific value of his results and viability of Daniel Dombek as a young researcher in the scientific community.

I fully recommend to award Daniel Dombek the PhD degree.

July 20, 2014

Zuzana Masáková