université PARIS DIDEROT PARIS 7

LABORATOIRE
D'INFORMATIQUE
ALGORITHMIQUE :
FONDEMENTS ET
APPLICATIONS

cnrs

Christiane Frougny
Emeritus Professor

Paris, le 11 juillet 2014

Report on the PhD thesis
Non-standard representations of numbers
submitted by
Daniel DOMBEK

This thesis is devoted to the study of the representation of numbers, more precisely to two different topics, the positional representations of real numbers with negative real base and the possibility of representing algebraic integers in a given number field by means of algebraic units.

The theory of β-expansions, where β is a positive real number, has been introduced by Rényi in 1957. It is a generalization of the classical representation of numbers in base 10 or, more generally, in a positive integer base. This domain has been largely studied, and has many connections with number theory, ergodic theory, words combinatorics, automata theory, discrete mathematics, algorithms and computer arithmetic.

The so-called $(-\beta)$-expansions, were introduced by Ito and Sadahiro in 2009 as an analogue to the β-expansions of Rényi. Note that the case of negative integer base goes back to Grünwald in a paper of 1885. The first part of this thesis consists in showing how the properties satisfied in the positive case extend or not to the negative case.

Daniel DOMBEK starts by proposing a generalization of $(-\beta)$-expansions and studies its properties, with the emphasis on deciding the admissibility of digit strings. This generalization is motivated by the fact that the case of negative base is much more tricky than the case of positive base : the shift-invariance and the non-triviality of the set of finite $(-\beta)$-expansions are not always guaranteed. But the result of Schmidt in positive base can be extended : if the $(-\beta)$-expansion of every rational number is eventually periodic, then β is a Pisot or a Salem number.

The set $\mathbb{Z}_{-\beta}$ of $(-\beta)$-integers is composed of the numbers which have an expansion formed only with non-negative powers of the base $(-\beta)$. Daniel DOMBEK provides the description of distances between neighbors in $\mathbb{Z}_{-\beta}$ and shows that there exists an encoding of $\mathbb{Z}_{-\beta}$, by an infinite word which can be generated by iterating an antimorphism.

LIAFA - CNRS UMR 7089 - Université Paris-Diderot - Case 7014 - F-75205 Paris Cedex 13
Tél. : +33 1 57 27 92 26 - Fax : +33 1 57 27 94 09
Courriel : Christiane.Frougny@liafa.univ-paris-diderot.fr
URL : http://www.liafa.univ-paris-diderot.fr/~cf

LABORATOIRE
D'INFORMATIQUE
ALGORITHMIQUE :
FONDEMENTS ET
APPLICATIONS

In positive base, it is known that the spectrum of β is equal to the set of β-integers if and only if β is a confluent Parry number. Daniel DOMBEK proves that a similar (but not exactly the same) result holds true in the negative base case. Moreover he answers the delicate question of the similarity of the sets of β- and $(-\beta)$-integers. A complete characterization of the distances is given in the case of Pisot numbers of low degree. Some nice tilings of the plane are also presented.

In the second part of this work the author starts with providing the upper bound on the length of arithmetic progressions of integers with bounded norm in modulus. He then generalizes the so-called unit sum number problem, the problem of determining whether all algebraic integers of a given number field can be expressed as sums of units. He replaces sums of units with either sums of integers of bounded field norm or with linear combinations of units with rational coefficients. He characterizes under which conditions are these representations indeed possible. Finally, he works with the notion of DUG-fields, which are fields with the property that all algebraic integers can be expressed as sums of distinct units. Then by using new methods he is able to extend the list of totally complex quartic fields which are DUG. What is particularly interesting is that some of the results are obtained by considering numeration in a base which is a complex Pisot number.

I have particularly appreciated the fact that, although the results are published in collaboration with several authors, Daniel DOMBEK has completely rewritten everything with a unified presentation. Moreover the state of the art is totally covered, and will be very useful to other researchers. The proofs are generally quite involved. There are also many examples.

The methods used in the thesis are a clever mixture of words combinatorics, discrete mathematics concepts, such as substitutions, as well as classical tools from algebra and number theory. In the conclusion the author lists a number of interesting open questions.

Although I am a specialist only of the first topic of this thesis, there is no doubt in my mind that Daniel DOMBEK has a perfect knowledge of the two domains covered. By being able to work in these two quite different fields, and to produce new links between both, he makes the proof that he is an excellent scientist. His results have been published in seven publications in collaboration. So he deserves to become a Doctor.

Ch Frougny

Christiane Frougny
Emeritus Professor

LIAFA - CNRS UMR 7089 - Université Paris-Diderot - Case 7014 - F-75205 Paris Cedex 13
Tél. : +33 1 57 27 92 26 - Fax : +33 1 57 27 94 09
Courriel : Christiane.Frougny@liafa.univ-paris-diderot.fr
URL : http://www.liafa.univ-paris-diderot.fr/~cf